Beyond space and blocks: Generating networks with arbitrary structure

Rémy Cazabet[1], Jacques Fize[2], Salvatore Citraro[3], Giulio Rossetti[3]

1. Univ Lyon, UCBL, CNRS, INSA Lyon, LIRIS, UMR5205, F-69622 Villeurbanne, France

2. Univ Lyon, Lyon 2, ERIC UR 3083, Bron, 69676, France

3. Institute of Information Science and Technologies "A. Faedo" (ISTI), National Research Council (CNR), G. Moruzzi 1, Pisa, Italy

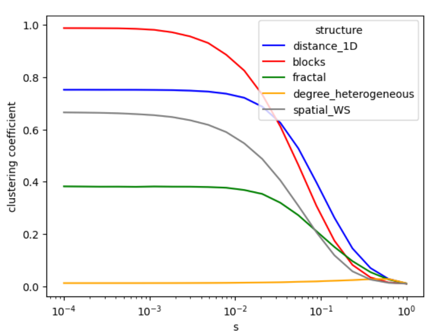
The mesoscale organization of networks is one of the most studied topics in network science. Some structures in particular have attracted a lot of attention, such as the block or community structure, the spatial structure, or the core-periphery structure. Many works have been published on how to detect these structures in observed networks, and how to generate random graphs having such a structure. In this work, we propose a framework to generate random graphs 1)having a desired number of nodes and edges, 2) following a desired structure –not limited to blocks and spatial structures 3) whose structure strength is controlled with a single parameter, from deterministic to fully random.

The principle is to define a function $f(u, v) = d, d \in \mathbb{R}$. d is used to rank pairs of nodes in their order of structure preference. A parametric function derived from Bézier curves is used to set the probability of observing each pair of edges among n nodes such as the expected number of edges equals m. m and n are chosen as objectives by the experimenter. The function itself is controlled by a structure strength parameter $s \in [0, 1]$ such as when $s = 1$, all node pairs have the same probability to be connected (ER random graph); when $s = 0$, the m edges connect the m node pairs of highest d, and the probabilities to observe an edge given the rank in d interpolates smoothly for values in-between.

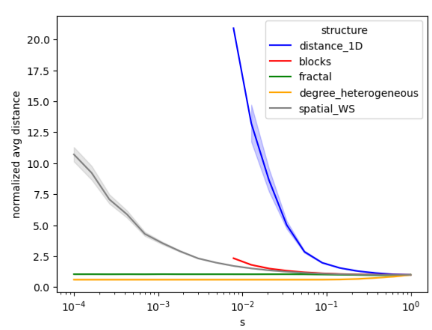
To exemplify an application of this framework, we reproduce a classic experiment in network science, the existence of a *small-world regime* (high clustering coefficient, short average distance) when adding randomness to a network with regular structure, as observed by Watts and Strogatz[1]. We mimic this experiment by varying the strength of the parameter s, from fully structure-controlled to randomness.

We pick some different structure functions as examples of possibilities: 1)**spatialWS** is a structure intended to mimic the original Watts-Strogatz small world experiment, i.e., $f(u, v) = 1$ if $|u - v| \mod (n - (\hat{k}/2)) <= \hat{k}/2$, else 0, 2)**spatial(1D)**: $f(u, v) = |l(u) - l(v)|$, where $l(u)$ reflects the position of u, 3)**Assortative Blocks**: $f(u, v) = \delta_{c(u),c(v)}$, with $c(u)$ the block of u and δ the Kronecker Delta, with the number of blocks chosen such as block sizes=ceil($\hat{k}$) 4)**Degree heterogeneous**, $f(u, v) = u * n + v$, where u, v are consecutive node indices $\in [0, n - 1]$, 5)A custom structure called **fractal graph**, in which nodes are assigned in a balanced binary tree t, and $f(u, v) = d_t(u, v)$, with $d_t(u, v)$ the distance between u and v in the tree.

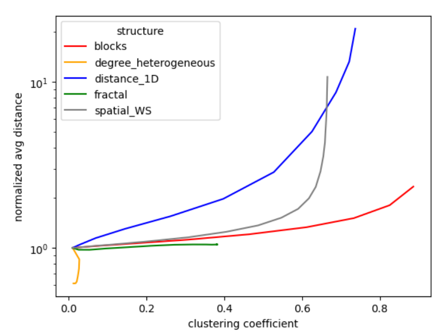
As in the original WS experiment, we pick $m = 1000, n = 5000$. We observe the expected well marked Small-World regime in SpatialWS, while it is absent in degree heterogeneous, and less marked in distance1D. It seems on the contrary stronger in the chosen block structure(although the average distance is not defined for small values due to disconnected blocks). The proposed fractal approach seems to always be in the Small-World regime, although never reaching very large clustering coefficient.



(a) Clustering coefficient



(b) Average distance



(c) Small-World plot

References

[1] Duncan J Watts and Steven H Strogatz. Collective dynamics of 'small-world'networks. *nature*, 393(6684):440–442, 1998.